OCR RECOGNISING ACHIEVEMENT

Friday 20 January 2012 – Afternoon

A2 GCE MATHEMATICS (MEI)

4753/01 Methods for Advanced Mathematics (C3)

QUESTION PAPER

Candidates answer on the Printed Answer Book.

OCR supplied materials:
- Printed Answer Book 4753/01
- MEI Examination Formulae and Tables (MF2)

Other materials required:
- Scientific or graphical calculator

Duration: 1 hour 30 minutes

4733100112

INSTRUCTIONS TO CANDIDATES

These instructions are the same on the Printed Answer Book and the Question Paper.
- The Question Paper will be found in the centre of the Printed Answer Book.
- Write your name, centre number and candidate number in the spaces provided on the Printed Answer Book. Please write clearly and in capital letters.
- **Write your answer to each question in the space provided in the Printed Answer Book**. Additional paper may be used if necessary but you must clearly show your candidate number, centre number and question number(s).
- Use black ink. HB pencil may be used for graphs and diagrams only.
- Read each question carefully. Make sure you know what you have to do before starting your answer.
- Answer **all** the questions.
- Do **not** write in the bar codes.
- You are permitted to use a scientific or graphical calculator in this paper.
- Final answers should be given to a degree of accuracy appropriate to the context.

INFORMATION FOR CANDIDATES

This information is the same on the Printed Answer Book and the Question Paper.
- The number of marks is given in brackets **[ ]** at the end of each question or part question on the Question Paper.
- You are advised that an answer may receive **no marks** unless you show sufficient detail of the working to indicate that a correct method is being used.
- The total number of marks for this paper is **72**.
- The Printed Answer Book consists of **16** pages. The Question Paper consists of **8** pages. Any blank pages are indicated.

INSTRUCTION TO EXAMS OFFICER / INVIGILATOR
- Do not send this Question Paper for marking; it should be retained in the centre or recycled. Please contact OCR Copyright should you wish to re-use this document.

Section A (36 marks)

1 Differentiate $x^2 \tan 2x$. **[3]**

2 The functions f(x) and g(x) are defined as follows.

$$f(x) = \ln x, \quad x > 0$$
$$g(x) = 1 + x^2, \quad x \in \mathbb{R}$$

Write down the functions fg(x) and gf(x), and state whether these functions are odd, even or neither. **[4]**

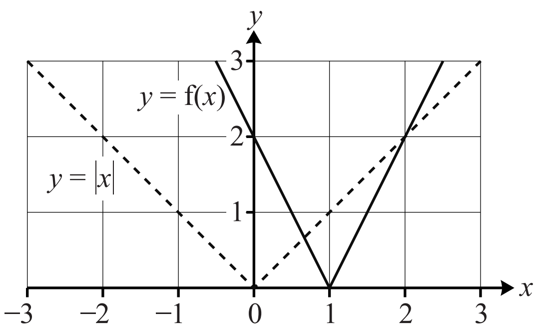
3 Show that $\int_0^{\frac{\pi}{2}} x \cos \tfrac{1}{2}x \, dx = \frac{\sqrt{2}}{2}\pi + 2\sqrt{2} - 4$. **[5]**

4 Prove or disprove the following statement:

'No cube of an integer has 2 as its units digit.' **[2]**

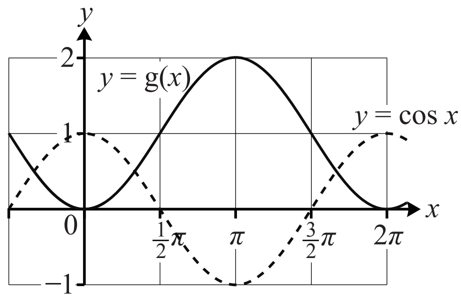
5 Each of the graphs of $y = f(x)$ and $y = g(x)$ below is obtained using a sequence of two transformations applied to the corresponding dashed graph. In each case, state suitable transformations, and hence find expressions for f(x) and g(x).

(i)



[3]

(ii)



[3]

6 Oil is leaking into the sea from a pipeline, creating a circular oil slick. The radius r metres of the oil slick t hours after the start of the leak is modelled by the equation

$$r = 20(1 - e^{-0.2t}).$$

(i) Find the radius of the slick when $t = 2$, and the rate at which the radius is increasing at this time. **[4]**

(ii) Find the rate at which the area of the slick is increasing when $t = 2$. **[4]**

7 Fig. 7 shows the curve $x^3 + y^3 = 3xy$. The point P is a turning point of the curve.

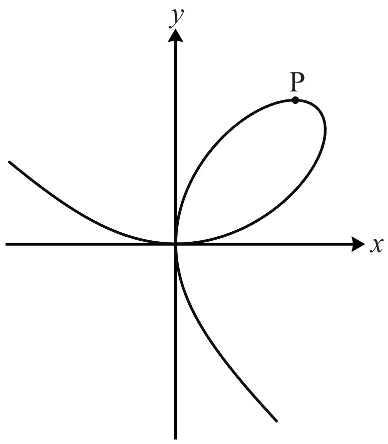


Fig. 7

(i) Show that $\frac{dy}{dx} = \frac{y - x^2}{y^2 - x}$. **[4]**

(ii) Hence find the exact x-coordinate of P. **[4]**

Section B (36 marks)

8 Fig. 8 shows the curve $y = \frac{x}{\sqrt{x-2}}$, together with the lines $y = x$ and $x = 11$.
The curve meets these lines at P and Q respectively. R is the point (11, 11).

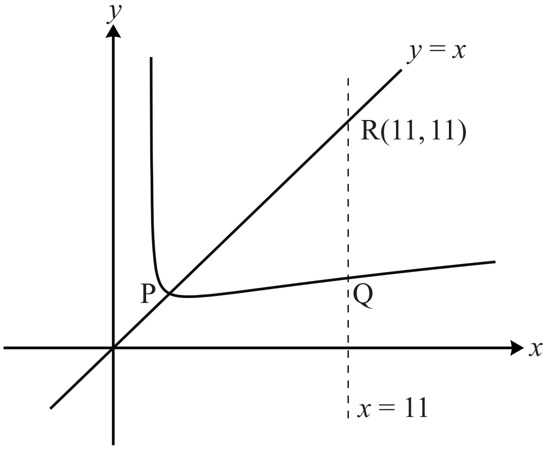


Fig. 8

(i) Verify that the x-coordinate of P is 3. **[2]**

(ii) Show that, for the curve, $\frac{dy}{dx} = \frac{x-4}{2(x-2)^{\frac{3}{2}}}$.

Hence find the gradient of the curve at P. Use the result to show that the curve is **not** symmetrical about $y = x$. **[7]**

(iii) Using the substitution $u = x - 2$, show that $\int_3^{11} \frac{x}{\sqrt{x-2}}\, dx = 25\frac{1}{3}$.

Hence find the area of the region PQR bounded by the curve and the lines $y = x$ and $x = 11$. **[9]**

9 Fig. 9 shows the curves $y = \mathrm{f}(x)$ and $y = \mathrm{g}(x)$. The function $y = \mathrm{f}(x)$ is given by

$$\mathrm{f}(x) = \ln\left(\frac{2x}{1+x}\right), \ x > 0.$$

The curve $y = \mathrm{f}(x)$ crosses the x-axis at P, and the line $x = 2$ at Q.

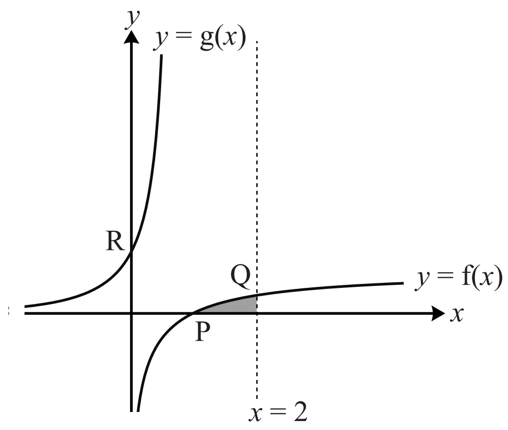


Fig. 9

(i) Verify that the x-coordinate of P is 1.

Find the exact y-coordinate of Q. **[2]**

(ii) Find the gradient of the curve at P. [Hint: use $\ln\frac{a}{b} = \ln a - \ln b$.] **[4]**

The function $\mathrm{g}(x)$ is given by

$$\mathrm{g}(x) = \frac{\mathrm{e}^x}{2 - \mathrm{e}^x}, \quad x < \ln 2.$$

The curve $y = \mathrm{g}(x)$ crosses the y-axis at the point R.

(iii) Show that $\mathrm{g}(x)$ is the inverse function of $\mathrm{f}(x)$.

Write down the gradient of $y = \mathrm{g}(x)$ at R. **[5]**

(iv) Show, using the substitution $u = 2 - \mathrm{e}^x$ or otherwise, that $\int_0^{\ln\frac{4}{3}} \mathrm{g}(x)\,\mathrm{d}x = \ln\frac{3}{2}$.

Using this result, show that the exact area of the shaded region shown in Fig. 9 is $\ln\frac{32}{27}$.
[Hint: consider its reflection in $y = x$.] **[7]**

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OCR
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Candidate forename		Candidate surname	

Centre number						Candidate number				

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Turn over

Section A (36 marks)

1	
2	

3

4	

PLEASE DO NOT WRITE IN THIS SPACE.

5 (i)	
5 (ii)	

6 (i)

6 (ii)

7 (i)	

7 (ii)

Section B (36 marks)

8 (i)	
8 (ii)	
	(answer space continued on next page)

8 (ii)	(continued)
8 (iii)	
	(answer space continued overleaf)

Turn over

8 (iii)	(continued)

9 (i)

9 (ii)

9 (iii)

9 (iv)

(answer space continued overleaf)

9 (iv)	(continued)

RECOGNISING ACHIEVEMENT

GCE

Mathematics (MEI)

Advanced GCE

Unit **4753:** Methods for Advanced Mathematics

Mark Scheme for January 2012

OCR (Oxford Cambridge and RSA) is a leading UK awarding body, providing a wide range of qualifications to meet the needs of candidates of all ages and abilities. OCR qualifications include AS/A Levels, Diplomas, GCSEs, OCR Nationals, Functional Skills, Key Skills, Entry Level qualifications, NVQs and vocational qualifications in areas such as IT, business, languages, teaching/training, administration and secretarial skills.

It is also responsible for developing new specifications to meet national requirements and the needs of students and teachers. OCR is a not-for-profit organisation; any surplus made is invested back into the establishment to help towards the development of qualifications and support, which keep pace with the changing needs of today's society.

This mark scheme is published as an aid to teachers and students, to indicate the requirements of the examination. It shows the basis on which marks were awarded by examiners. It does not indicate the details of the discussions which took place at an examiners' meeting before marking commenced.

All examiners are instructed that alternative correct answers and unexpected approaches in candidates' scripts must be given marks that fairly reflect the relevant knowledge and skills demonstrated.

Mark schemes should be read in conjunction with the published question papers and the report on the examination.

OCR will not enter into any discussion or correspondence in connection with this mark scheme.

Any enquiries about publications should be addressed to:

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Telephone: 0870 770 6622
Facsimile: 01223 552610
E-mail: publications@ocr.org.uk

Annotations and abbreviations

Annotation in scoris	Meaning
✓and ✗	
BOD	Benefit of doubt
FT	Follow through
ISW	Ignore subsequent working
M0, M1	Method mark awarded 0, 1
A0, A1	Accuracy mark awarded 0, 1
B0, B1	Independent mark awarded 0, 1
SC	Special case
^	Omission sign
MR	Misread
Highlighting	

Other abbreviations in mark scheme	Meaning
E1	Mark for explaining
U1	Mark for correct units
G1	Mark for a correct feature on a graph
M1 dep*	Method mark dependent on a previous mark, indicated by *
cao	Correct answer only
oe	Or equivalent
rot	Rounded or truncated
soi	Seen or implied
www	Without wrong working

Subject-specific Marking Instructions for GCE Mathematics (MEI) Pure strand

a Annotations should be used whenever appropriate during your marking.

The A, M and B annotations must be used on your standardisation scripts for responses that are not awarded either 0 or full marks. It is vital that you annotate standardisation scripts fully to show how the marks have been awarded.

For subsequent marking you must make it clear how you have arrived at the mark you have awarded.

b An element of professional judgement is required in the marking of any written paper. Remember that the mark scheme is designed to assist in marking incorrect solutions. Correct *solutions* leading to correct answers are awarded full marks but work must not be judged on the answer alone, and answers that are given in the question, especially, must be validly obtained; key steps in the working must always be looked at and anything unfamiliar must be investigated thoroughly.

Correct but unfamiliar or unexpected methods are often signalled by a correct result following an *apparently* incorrect method. Such work must be carefully assessed. When a candidate adopts a method which does not correspond to the mark scheme, award marks according to the spirit of the basic scheme; if you are in any doubt whatsoever (especially if several marks or candidates are involved) you should contact your Team Leader.

c The following types of marks are available.

M
A suitable method has been selected and *applied* in a manner which shows that the method is essentially understood. Method marks are not usually lost for numerical errors, algebraic slips or errors in units. However, it is not usually sufficient for a candidate just to indicate an intention of using some method or just to quote a formula; the formula or idea must be applied to the specific problem in hand, eg by substituting the relevant quantities into the formula. In some cases the nature of the errors allowed for the award of an M mark may be specified.

A
Accuracy mark, awarded for a correct answer or intermediate step correctly obtained. Accuracy marks cannot be given unless the associated Method mark is earned (or implied). Therefore M0 A1 cannot ever be awarded.

B
Mark for a correct result or statement independent of Method marks.

E
A given result is to be established or a result has to be explained. This usually requires more working or explanation than the establishment of an unknown result.

Unless otherwise indicated, marks once gained cannot subsequently be lost, eg wrong working following a correct form of answer is ignored. Sometimes this is reinforced in the mark scheme by the abbreviation isw. However, this would not apply to a case where a candidate passes through the correct answer as part of a wrong argument.

d When a part of a question has two or more 'method' steps, the M marks are in principle independent unless the scheme specifically says otherwise; and similarly where there are several B marks allocated. (The notation 'dep *' is used to indicate that a particular mark is dependent on an earlier, asterisked, mark in the scheme.) Of course, in practice it may happen that when a candidate has once gone wrong in a part of a question, the work from there on is worthless so that no more marks can sensibly be given. On the other hand, when two or more steps are successfully run together by the candidate, the earlier marks are implied and full credit must be given.

e The abbreviation ft implies that the A or B mark indicated is allowed for work correctly following on from previously incorrect results. Otherwise, A and B marks are given for correct work only — differences in notation are of course permitted. A (accuracy) marks are not given for answers obtained from incorrect working. When A or B marks are awarded for work at an intermediate stage of a solution, there may be various alternatives that are equally acceptable. In such cases, exactly what is acceptable will be detailed in the mark scheme rationale. If this is not the case please consult your Team Leader.

Sometimes the answer to one part of a question is used in a later part of the same question. In this case, A marks will often be 'follow through'. In such cases you must ensure that you refer back to the answer of the previous part question even if this is not shown within the image zone. You may find it easier to mark follow through questions candidate-by-candidate rather than question-by-question.

f Wrong or missing units in an answer should not lead to the loss of a mark unless the scheme specifically indicates otherwise. Candidates are expected to give numerical answers to an appropriate degree of accuracy, with 3 significant figures often being the norm. Small variations in the degree of accuracy to which an answer is given (eg 2 or 4 significant figures where 3 is expected) should not normally be penalised, while answers which are grossly over- or under-specified should normally result in the loss of a mark. The situation regarding any particular cases where the accuracy of the answer may be a marking issue should be detailed in the mark scheme rationale. If in doubt, contact your Team Leader.

g Rules for replaced work

If a candidate attempts a question more than once, and indicates which attempt he/she wishes to be marked, then examiners should do as the candidate requests.

If there are two or more attempts at a question which have not been crossed out, examiners should mark what appears to be the last (complete) attempt and ignore the others.

NB Follow these maths-specific instructions rather than those in the assessor handbook.

h For a *genuine* misreading (of numbers or symbols) which is such that the object and the difficulty of the question remain unaltered, mark according to the scheme but following through from the candidate's data. A penalty is then applied; 1 mark is generally appropriate, though this may differ for some units. This is achieved by withholding one A mark in the question.

Note that a miscopy of the candidate's own working is not a misread but an accuracy error.

Question			Answer	Marks	Guidance	
1			$y = x^2 \tan 2x$ $\Rightarrow$ dy/dx = $2x^2\sec^2 2x + 2x\tan 2x$ **OR** $y = x^2 \dfrac{\sin 2x}{\cos 2x}$ $\dfrac{dy}{dx} = x^2 \dfrac{\cos 2x.2\cos 2x - \sin 2x(-2\sin 2x)}{\cos^2 2x} + 2x\dfrac{\sin 2x}{\cos 2x}$ $= \ldots = 2x^2\sec^2 2x + 2x\tan 2x$ **OR** $y = \dfrac{x^2 \sin 2x}{\cos 2x}$ $\dfrac{dy}{dx} = \dfrac{\cos 2x(2x\sin 2x + x^2 2\cos 2x) - 2x^2 \sin 2x(-\sin 2x)}{\cos^2 2x}$ $= \ldots = 2x^2\sec^2 2x + 2x\tan 2x$	M1 M1 A1cao M1 A1 A1cao M1 A1 A1cao **[3]**	product rule d/du(tan u) = $\sec^2 u$ soi or $2x^2/\cos^2 2x + 2x\tan 2x$ product rule correct expression or $2x^2/\cos^2 2x + 2x\tan 2x$ (isw) quotient rule correct expression or $2x^2/\cos^2 2x + 2x\tan 2x$ (isw)	u × their v' + v × their u' attempted M0 if d/dx (tan 2x) =(2) $\sec^2 x$ isw *see additional notes for complete solution* u × their v' + v × their u' attempted or $(2x^2 + 2x\sin 2x\cos 2x)/\cos^2 2x$ or $2x^2/\cos^2 2x + 2x\sin 2x / \cos 2x$ *see additional notes for complete solution* (v × their u' − u × their v')/v^2 attempted or $(2x^2 + 2x\sin 2x\cos 2x)/\cos^2 2x$ or $2x^2/\cos^2 2x + 2x\sin 2x / \cos 2x$
2			fg(x) = $\ln(1+x^2)$ ($x \in \Re$) gf(x) = $1+(\ln x)^2$ ($x > 0$) $\ln(1+x^2)$ even $1 + (\ln x)^2$ neither	B1 B1 B1 B1 **[4]**	condone missing bracket, and missing or incorrect domains Penalise missing bracket Penalise missing bracket	If fg and gf the wrong way round, B1B0 not $1 + \ln(x^2)$
3			$u = x$, du/dx = 1, dv/dx = cos ½ x, v = 2sin ½ x $\int_0^{\pi/2} x\cos\frac{1}{2}x\,dx = \left[2x\sin\frac{1}{2}x\right]_0^{\pi/2} - \int_0^{\pi/2} 2\sin\frac{1}{2}x\,dx$ $= \left[2x\sin\frac{1}{2}x + 4\cos\frac{1}{2}x\right]_0^{\pi/2}$ $= \pi\sin\frac{\pi}{4} + 4\cos\frac{\pi}{4} - (2.0.\sin 0 + 4\cos 0)$ $= \pi.\frac{1}{\sqrt{2}} + 4.\frac{1}{\sqrt{2}} - 4$ $= \frac{\sqrt{2}}{2}\pi + 2\sqrt{2} - 4$*	M1 A1ft A1 M1 A1cao **[5]**	correct u, u', v, v' consistent with their u, v 2x sin ½ x + 4 cos ½ x oe (no ft) substituting correct limits into correct expression **NB AG**	but allow v to be any multiple of sin ½ x M0 if u = cos ½ x, , v' = x can be implied by one correct intermediate step

4			Cubes are 1, 8, 27, 64, 125, 216, 343, 512 [so false as] $8^3 = 512$	M1 A1 **[2]**	Attempt to find counter example counter-example identified (e.g. underlining, circling) [counter-examples all have 8 as units digit]	if no counter-example found, award M1 if at least 3 cubes are calculated. condone not explicitly stating statement is false
5	(i)		(One-way) stretch in y-direction, s.f. 2 or in x-direction s.f. ½ translation 1 to right (2 if followed by x-stretch) $y = 2\|x-1\|$	 B1 B1 B1 **[3]**	 must specify s.f. and direction o.e. e.g. $y = \|2x-2\|$ $y = \|2(x-1)\|$	Allow 'compress', 'squeeze'(for s.f. ½), but not 'enlarge', 'x-coordinates halved', etc Allow 'shift','move' or vector only, 'right 1' Don't allow misreads (e.g. transforming solid graph to dashed graph) Award B1 for one of these seen, and a second B1 if combined transformations are correct
5	(ii)		Reflection in x-axis or translation right $\pm\pi$ or rotation of 180° [about O] translation +1 in y-direction (− 1 if followed by reflection in x-axis $y = 1 - \cos x$	 B1 B1 B1 **[3]**	 $\begin{pmatrix}\pm\pi\\1\end{pmatrix}$ is B2 allow $1 + \cos(x \pm\pi)$ (bracket needed)	Translations as above. Reflection: must specify axis, allow 'flip' Rotation: condone no origin stated. *See additional notes for other possible solutions*. Award B1 for any one of these seen, and a second B1 if combined transformations are correct
6	(i)		When $t = 2$, $r = 20(1 - e^{-0.4}) = 6.59$ m $dr/dt = -20 \times (-0.2e^{-0.2t})$ $= 4e^{-0.2t}$ When $t = 2$, $dr/dt = 2.68$	M1A1 M1 A1 **[4]**	6.6 or art 6.59 $-0.2e^{-0.2t}$ soi 2.7 or art 2.68 or $4e^{-0.4}$	 mark final answer
6	(ii)		$A = \pi r^2$ $\Rightarrow$ $dA/dr = 2\pi r$ (= 41.428…) $dA/dt = (dA/dr) \times (dr/dt)$ = 41.428… × 2.68 = 111 m^2/hr	M1 A1 M1 A1 **[4]**	attempt to differentiate πr^2 $dA/dr = 2\pi r$ (not dA/dt, dr/dA etc) (o.e.) chain rule expressed in terms of their A, r or implied 110 or art 111	or differentiating $400\pi(1 - e^{-0.2t})^2$ M1 $dA/dt = 400\pi.2(1 - e^{-0.2t}).(-0.2e^{-0.2t})$ A1 substitute $t = 2$ into correct dA/dt M1 (Could use another letter for A)

7	(i)		$x^3 + y^3 = 3xy$ $\Rightarrow \quad 3x^2 + 3y^2(\mathrm{d}y/\mathrm{d}x) = 3x(\mathrm{d}y/\mathrm{d}x) + 3y$ $\Rightarrow \quad (3y^2 - 3x)(\mathrm{d}y/\mathrm{d}x) = 3y - 3x^2$ $\Rightarrow \quad \mathrm{d}y/\mathrm{d}x = (3y - 3x^2)/(3y^2 - 3x)$ $= (y - x^2)/(y^2 - x)$*	B1B1 M1 A1cao [4]	LHS, RHS Condone $3x\mathrm{d}y/\mathrm{d}x+y$ (i.e.with missing bracket) if recovered thereafter collecting terms in $\mathrm{d}y/\mathrm{d}x$ and factorising NB **AG**	or equivalent if re-arranged. ft correct algebra on incorrect expressions with two $\mathrm{d}y/\mathrm{d}x$ terms Ignore starting with '$\mathrm{d}y/\mathrm{d}x = \ldots$' unless pursued
7	(ii)		TP when $y - x^2 = 0$ $\Rightarrow \quad y = x^2$ $\Rightarrow \quad x^3 + x^6 = 3x.x^2$ $\Rightarrow \quad x^6 = 2x^3$ $\Rightarrow \quad x^3 = 2$ (or $x = 0$) $\Rightarrow \quad x = \sqrt[3]{2}$	 M1 M1 A1 A1cao [4]	 or $x = \sqrt{y}$ substituting for y in implicit eqn (allow one slip, e.g. x^5) o.e. (soi) must be exact	 or x for y (i.e. $y^{3/2} + y^3 = 3y^{1/2}y$ o.e.) or $y^{3/2} = 2$ $x = 1.2599\ldots$ is A0 (but can isw $x = \sqrt[3]{2}$)
8	(i)		When $x = 3$, $y = 3/\sqrt{(3 - 2)} = 3$ So P is (3, 3) which lies on $y = x$	M1 A1 **[2]**	substituting $x = 3$ (both x's) $y = 3$ and completion ('3 = 3' is enough)	or $x = x/\sqrt{(x - 2)}$ M1 $\Rightarrow x = 3$ A1(by solving or verifying)
8	(ii)		$\dfrac{\mathrm{d}y}{\mathrm{d}x} = \dfrac{\sqrt{x-2}.1 - x.\frac{1}{2}.(x-2)^{-1/2}}{x-2}$ $= \dfrac{x-2-\frac{1}{2}x}{(x-2)^{3/2}} = \dfrac{\frac{1}{2}x-2}{(x-2)^{3/2}}$ $= \dfrac{x-4}{2(x-2)^{3/2}}$* When $x = 3$, $\mathrm{d}y/\mathrm{d}x = -\tfrac{1}{2} \times 1^{3/2}$ $= -\tfrac{1}{2}$ This gradient would be -1 if curve were symmetrical about $y = x$	M1 A1 M1 A1 M1 A1 A1cao **[7]**	Quotient or product rule PR: $-\tfrac{1}{2}x(x - 2)^{-3/2} + (x - 2)^{-1/2}$ correct expression × top and bottom by $\sqrt{(x - 2)}$ o.e. e.g. taking out factor of $(x - 2)^{-3/2}$ NB **AG** substituting $x = 3$ or an equivalent valid argument	If correct formula stated, allow one error; otherwise QR must be on correct u and v, with numerator consistent with their derivatives and denominator correct initially allow ft on correct equivalent algebra from their incorrect expression

8	(iii)		$u = x - 2 \Rightarrow du/dx = 1 \Rightarrow du = dx$ When $x = 3$, $u = 1$ when $x = 11$, $u = 9$ $\Rightarrow \int_3^{11} \frac{x}{\sqrt{x-2}}dx = \int_1^9 \frac{u+2}{u^{1/2}}du$ $= \int_1^9 (u^{1/2} + 2u^{-1/2})du$ $= \left[\frac{2}{3}u^{3/2} + 4u^{1/2}\right]_1^9$ $= (18 + 12) - (2/3 + 4)$ $= 25\frac{1}{3}$ * Area under $y = x$ is ½ (3 + 11) × 8 = 56 Area = (area under $y = x$) – (area under curve) so required area $= 56 - 25\frac{1}{3} = 30\frac{2}{3}$	B1 B1 M1 A1 M1 A1cao B1 M1 A1cao **[9]**	or $dx/du = 1$ $\int \frac{u+2}{u^{1/2}}(du)$ splitting their fraction (correctly) and $u/u^{1/2} = u^{1/2}$ (or $\sqrt{u}$) $\left[\frac{2}{3}u^{3/2} + 4u^{1/2}\right]$ (o.e) substituting correct limits NB AG o.e. (e.g. 60.5 – 4.5) soi from working 30.7 or better	No credit for integrating initial integral by parts. Condone $du = 1$. Condone missing du's in subsequent working. or integration by parts: $2u^{1/2}(u+2) - \int 2u^{1/2}du$ (must be fully correct – condone missing bracket by parts: $[2u^{1/2}(u+2) - 4u^{3/2}/3]$ F(9) – F(1) (u) or F(11) – F(3) (x) dep substitution and integration attempted must be trapezium area: $60.5 - 25\frac{1}{3}$ is M0
9	(i)		When $x = 1$, f(1) = ln(2/2) = ln 1 = 0 so P is (1, 0) f(2) = ln(4/3)	B1 B1 **[2]**	or $\ln(2x/1+x) = 0 \Rightarrow 2x/(1+x) = 1 \Rightarrow 2x = 1+x \Rightarrow x = 1$	if approximated, can isw after ln(4/3)
9	(ii)		$y = \ln(2x) - \ln(1 + x)$ $\Rightarrow \frac{dy}{dx} = \frac{2}{2x} - \frac{1}{1+x}$ **OR** $\frac{d}{dx}(\frac{2x}{1+x}) = \frac{(1+x)2 - 2x.1}{(1+x)^2} = \frac{2}{(1+x)^2}$ $\frac{dy}{dx} = \frac{2}{(1+x)^2}.\frac{1}{2x/(1+x)} = \frac{1}{x(1+x)}$ At P, $dy/dx = 1 - ½ = ½$	M1 M1 A1cao B1 M1 A1 A1cao **[4]**	one term correct mark final ans correct quotient or product rule chain rule attempted o.e., but mark final ans	condone lack of brackets $2/2x$ or $-1/(1+x)$ need not be simplified need not be simplified

9	(iii)		$x = \ln[2y/(1+y)]$ or $\Rightarrow e^x = 2y/(1+y)$ $\Rightarrow e^x(1+y) = 2y$ $\Rightarrow e^x = 2y - e^x y = y(2 - e^x)$ $\Rightarrow y = e^x/(2 - e^x)$ [= g(x)] **OR** $gf(x)=g(2x/(1+x)) = e^{\ln[2x/(1+x)]}/\{2-e^{\ln[2x/(1+x)]}\}$ $= \dfrac{2x/(1+x)}{2-2x/(1+x)}$ $= \dfrac{2x}{2+2x-2x} = \dfrac{2x}{2} = x$ gradient at R = 1/ ½ = 2	 B1 B1 B1 B1 M1 A1 M1A1 B1 ft **[5]**	($x \leftrightarrow y$ here or at end to complete) completion forming gf or fg 1/their ans in (ii) unless ±1 or 0	$x = e^y/(2 - e^y)$ $x(2 - e^y) = e^y$ B1 $2x = e^y + xe^y = e^y(1 + x)$ B1 $2x/(1+x) = e^y$ B1 $\ln[2x/(1+x)] = y$ [= f(x)] B1 $fg(x) = \ln\{2e^x/(2-e^x)/[1+ e^x/(2-e^x)]\}$ M1 $= \ln[2e^x/(2 - e^x + e^x)]$ A1 $= \ln(e^x) = x$ M1A1 2 must follow ½ for 9(ii) unless g′(x) used (*see additional notes*)
9	(iv)		let $u = 2 - e^x \Rightarrow du/dx = -e^x$ $x = 0, u = 1, x = \ln(4/3), u = 2 - 4/3 = 2/3$ $\Rightarrow \int_0^{\ln(4/3)} g(x)\,dx = \int_1^{2/3} -\frac{1}{u}\,du$ $= [-\ln(u)]_1^{2/3} = -\ln(2/3) + \ln 1 = \ln(3/2)$* Shaded region = rectangle – integral = 2ln(4/3) – ln(3/2) = ln(16/9 × 2/3) = ln(32/27)*	 B1 M1 A1 A1cao M1 B1 A1cao **[7]**	$2-e^0 = 1$, and $2 - e^{\ln(4/3)} = 2/3$ seen $\int -1/u\,du$ condone $\int 1/u\,du$ [–ln(u)] (could be [ln*u*] if limits swapped) NB AG rectangle area = 2ln(4/3) NB AG must show at least one step from 2ln(4/3) – ln(3/2)	here or later (i.e. after substituting 0 and ln(4/3) into $\ln(2 - e^x)$) or by inspection [$k \ln(2 - e^x)$] $k = -1$ Allow full marks here for correctly evaluating $\int_1^2 \ln(\frac{2x}{1+x})\,dx$ (*see additional notes*)

Additional notes and solutions

1. $y = x^2 \dfrac{\sin 2x}{\cos 2x}$ $\dfrac{dy}{dx} = x^2 \dfrac{\cos 2x.2\cos 2x - \sin 2x(-2\sin 2x)}{\cos^2 2x} + 2x\dfrac{\sin 2x}{\cos 2x} = x^2 \dfrac{2\cos^2 2x + 2\sin^2 2x}{\cos^2 2x} + 2x\dfrac{\sin 2x}{\cos 2x}$

$= x^2 \dfrac{2}{\cos^2 2x} + 2x\dfrac{\sin 2x}{\cos 2x} = 2x^2 \sec^2 2x + 2x\tan 2x$

$y = \dfrac{x^2 \sin 2x}{\cos 2x}$ $\dfrac{dy}{dx} = \dfrac{\cos 2x(2x\sin 2x + x^2 2\cos 2x) - 2x^2 \sin 2x(-\sin 2x)}{\cos^2 2x}$

$= \dfrac{2x\cos 2x\sin 2x + 2x^2\cos^2 2x - x^2 \sin 2x(-2\sin^2 2x)}{\cos^2 2x} = \dfrac{2x\cos 2x\sin 2x + 2x^2\cos^2 2x + 2x^2\sin^2 2x}{\cos^2 2x}$

$= \dfrac{2x\cos 2x\sin 2x + 2x^2}{\cos^2 2x} = 2x\tan 2x + 2x^2\sec^2 x$

5 (ii) translation $\begin{pmatrix}0\\1\end{pmatrix}$ then translation $\begin{pmatrix}\pm\pi\\0\end{pmatrix}$ translation $\begin{pmatrix}\pm\pi\\0\end{pmatrix}$ then translation $\begin{pmatrix}0\\1\end{pmatrix}$ translation $\begin{pmatrix}\pm\pi\\1\end{pmatrix}$ B2

translation $\begin{pmatrix}0\\1\end{pmatrix}$ then reflection in $y = 1$ translation $\begin{pmatrix}0\\1\end{pmatrix}$ then reflection in $x = ½\,\pi$ translation $\begin{pmatrix}0\\-1\end{pmatrix}$ then reflection in x-axis

reflection in x-axis then translation $\begin{pmatrix}0\\1\end{pmatrix}$ reflection in $y = 1$ then translation $\begin{pmatrix}0\\-1\end{pmatrix}$ rotation 180° about O then translation $\begin{pmatrix}0\\1\end{pmatrix}$

reflection in $y = ½$ B2

9(iii) last part: $g(x) = e^x/(2 - e^x) \Rightarrow g'(x) = [(2 - e^x)e^x - e^x(-e^x)]/(2 - e^x)^2 = 2e^x/(2 - e^x)^2$
or $g'(x) = e^x(-1)(-e^x)](2 - e^x)^{-2} + e^x(2 - e^x)^{-1}$
$g'(0) = 2.1/1^2 = 2$ B1

9(iv) last part

$\int_1^2 \ln(\dfrac{2x}{1+x})\,dx = \int_1^2 (\ln 2 + \ln x - \ln(1+x))\,dx = \left[x\ln 2 + x\ln x - x - (1+x)\ln(1+x) + x\right]_1^2$

$= 2\ln 2 + 2\ln 2 - 2 - 3\ln 3 + 2 - (\ln 2 - 1 - 2\ln 2 + 1) = 5\ln 2 - 3\ln 3 = \ln(32/27)$

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4753 Methods for Advanced Mathematics (Written Examination)

General Comments

The paper proved to be a good, fair test of candidates' attainment. All but the very weakest candidates managed to accumulate over 20 marks, and over 70% of candidates gained over half the marks. Getting over 65 marks was rare, however, and there were a number of quite demanding tests for the more able candidates. Virtually all candidates attempted all the questions and part questions. The usual variability of presentation, algebraic fluency (use of brackets, etc.) and accurate use of notation was evident.

It might be helpful to advise candidates that the answer booklets are designed to provide ample space for answers, and they should not worry if they fail to fill the space available. They should also be made aware that, in the case of offering more than one attempt at a solution, it is the last complete attempt which is marked, not the best. Sometimes this cost candidates marks – it is worth their while to indicate which attempt they wish to be marked.

One aspect of the syllabus which might be worth drawing specific attention to is transformations and their specification. Students should be encouraged to use the words *translation* (not 'move', 'shift', etc., or vector only), *one-way stretch* (not 'squash', 'squeeze', etc.), and *reflection* (not 'flip'). Descriptions which refer to coordinates (e.g. *y*-coordinates are doubled') score no marks. In fact, many of these descriptions were actually condoned in this paper, but in general will not be allowed.

Comments on Individual Questions

Section A

1 The derivative of tan x was usually familiar, but those candidates who started with sin 2x/cos 2x usually got lost in algebraic complexity. A surprising number lost marks through giving the derivative of tan $2x$ as $\sec^2 x$, or omitting the '2' in $2\sec^2 2x$. However, better candidates just wrote the result down.

2 This question was often well done. Marks were lost through omitting essential brackets, and stating that $1 + \ln x^2 = 1 + 2\ln x$. Very occasionally, fg and gf were the wrong way round.

3 There was a mixed response to the question, with plenty of faultless answers, but others with errors in $v = 2\sin \frac{1}{2} x$, e.g. $v = \sin \frac{1}{2} x$ or $-2 \sin \frac{1}{2} x$ or $\frac{1}{2} \sin \frac{1}{2} x$. Occasionally there was insufficient working to show that the given result had been established: candidates are well advised to include ample working.

4 This simple two-mark question was well answered, with the majority of candidates correctly identifying the counter-example $8^3 = 512$. Some candidates, however, did not understand what was meant by 'units digit'.

5 Candidates achieved mixed success here, with part (ii) answered a little better than part (i). Unlike in recent papers, we condoned inaccurately specified transformations, as the spirit of the question was to deduce the formula for the transformed function. In part (i), quite a few used the *x*-stretch after translating one to the right (instead of before). One-way stretching in the *x*-direction seemed to be more popular than in the *y*-direction. The form of the final function was often incorrect.

In part (ii), successful candidates were equally split between using a reflection in O*x* (sometimes described as a one way stretch in the *y*-direction with scale factor −1) and a translation of π in the *x*−direction. The final function was a little more successfully done.

6 In part (i), the first two marks for finding the radius when $t = 2$ were readily achieved. Not so the next two, with some generally rather poor attempts to differentiate $20(1 – e^{-0.2t})$. Quite a few candidates substituted $t = 2$ into $e^{-0.2t}$ to get $e^{-0.4}$, then differentiated this as $-0.4e^{-0.4}$. Some simply divided their value of *r* by 2.

Part (ii) offered some accessible marks for stating the chain rule, and for $dA/dr = 2\pi r$. The final mark depended on getting $dr/dt = 2.68$ from part (i).

7 Part (i) was very well done – it is pleasing to see how well implicit differentiation is understood, and the algebra to derive the given result was generally done well.

In part (ii), many fully correct answers notwithstanding, some failed to get beyond the first M1 for $y = x^2$; others who substituted for *y* in the implicit function sometimes erred with $(x^2)^3 = x^5$.

8 Part (i) was an easy two marks for nearly all candidates. However, sometimes it was difficult to tell whether it was made clear that the point (3, 3) lies on the line $y = x$.

In part (ii), both the product and quotient rules were seen – perhaps the product rule is slightly easier to sort out in this case. Although most gained the initial M1A1 for this, the algebra required to derive the given answer, either by using a common denominator or factoring out $(x – 2)^{-½}$, was poorly done. Most candidates should have been able to recover to get the derivative at $x = 3$, and 4/7 was a common mark for the part. The final mark, using this result to examine the symmetry of the function, was the preserve of more able candidates. Many thought that the P had to be a turning point for the graph to be symmetrical about $y = x$.

Part (iii) achieved mixed success. It was pleasing to see that most gained the B1 for $du = dx$; most got the second B1 for $(u + 2)\sqrt{u}$; thereafter, the 'M' for splitting the fraction was often lost – some used integration by parts here with some success (a sledgehammer to crack a nut?). Those who got beyond this hurdle often gained all 6 marks. The final 3 marks were often omitted, but the best candidates got all 9 marks; the most common error here was to use the triangle with vertices (0, 0), (11, 0) and (11, 11) rather than the trapezium formed by removing the triangle with vertices (0, 0), (3, 0) and (3, 3).

9 Part (i) offered two straightforward marks. Many approximated for ln (4/3), but we ignored this in subsequent working.

In part (ii), the hint proved valuable and was taken by nearly all candidates. However, many found the derivative of ln(2*x*) as 1/(2*x*) and lost two marks. Those who avoided this error usually scored all 4 marks.

Inverting the function in part (iii) was less successful than usual. This might have been caused by candidates using the 'hint' from the previous part to write $x = \ln 2y - \ln(1 + y)$, and then getting stuck. The gradient in the last part as the reciprocal of that in part (ii) was better answered than in previous papers.

Finally, part (iv) was the least well answered question. The new 'u' limits of 1 and 2/3 were usually present, but many lost the minus sign from $du = -e^{-x}dx$, and few gave fully convincing 'shows'. The last result was rarely done, though it was not possible to gather whether this was due to difficulty or lack of time.